weightier than Jones minor. In other words, in practical affairs we emphatically do not (and never shall) use the razor to cut away our tables and chairs and numbers and electrons. Ockham's razor is on the contrary an impractical maxim only applied in those moments of critical analysis when we desire to know what errors we are daily committing from pragmatic motives.

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THE LOGICAL SIGNIFICANCE OF "OCKHAM'S RAZOR."

In adopting the principle of parsimony or "Ockham's razor" as the supreme maxim of methodology, Mr. Russell seems to have been guided by esthetic motives. To some of us, indeed, it seems to be more beautiful to start the development of a science in deductive form from three fundamental or "primitive" ideas instead of from four and to define all the rest in terms of these three, and even more beautiful to start from two primitive ideas. But there seems to be a *logical* basis for this preference, for the reduction in the number of primitive ideas implies a discovery of dependence between those ideas at first taken to be indefinable. I shall try to make this more precise.

Formally, the ideas of a deductive science, both the primitive ones and the ones that are defined in terms of them, may be regarded as the "unknowns" in an algebraic problem, between which equations, representing the definitions, subsist. This is at the basis of the algebra of logic. If the number of independent equations is equal to that of the unknowns, the unknowns can be wholly defined. This is the case with the science of arithmetic, according to modern ideas; for number and the other ideas with which arithmetic is concerned are all definable in terms of logic. But if the number of independent equations is less than that of the unknowns, some of the unknowns are indefinable, and in fact it may be more convenient to leave more of the unknowns undefined than are strictly necessary, and preserve the equations between these indefinables. In algebra we may find that, with the unknowns x, y, z, we know that $x=f(z)$ and $z=\phi(y)$, and consequently that y is strictly speaking the only indefinable; but it may be symbolically impossible

to *express* x as a function of y. Of course, we may have an indefinite number of defined ideas, and it is easy to see that this number does not affect the question as to how many indefinables there are.

For example, Whitehead and Russell, in their theory of deduction, have expressed *implication* and *joint assertion* of propositions in terms of *negation* and *disjunction*. Sheffer and Nicod have succeeded in defining all the above four ideas in terms of a new function of propositions p and q denoted by "p/q" and which can be defined in the system of Whitehead and Russell as "Either p or q is false." Thus, in the system of Whitehead and Russell we have five ideas of which two are indefinable, while in Sheffer's system we have five ideas of which one is indefinable. The latter is preferable because it discovers a relation between the two undefined ideas of Whitehead and Russell; though this relation does not appear capable, in our system of symbols, of being expressed without making use of Sheffer's indefinable. Thus the principle of parsimony appears, from a logical point of view, to be simply the maxim that logical analysis is to be carried as far as possible; and this is no more than Dedekind's maxim that what *can* be proved *is* to be proved.

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INDEPENDENCE PROOFS AND THE THEORY OF IMPLICATION.

In the January number Mr. Lenzen writes a criticism of the traditional symbolic logic as exemplified by the system of Whitehead and Russell's *Principia Mathematica*. His criticism is that it fails to give a correct theory of mathematical deduction. The attempt is made to prove that, in certain cases, of two mathematically independent postulates one implies the other according to the method of the *Principia*.

The present writer was, at first, in sympathy with the criticism; but finally the error involved was detected. The chief illustration is taken from two independent postulates for algebra:

A2 $(a+b)+c=a+(b+c)$,
M5 $a\times b=b\times a$.